Deformable elastic structures

How theory can shape application

Through the latest advances in materials science, new deformable materials have emerged that are so advanced that they can't be easily described using existing mathematical theories. Dr Simon Eugster at the University of Stuttgart aims to show how the conception of new, more advanced theories can shape the application of these materials in the real world. His team hopes that their discoveries will soon have multiple real-world applications, including in the creation of 'soft robots' which are composed of highly deformable materials.

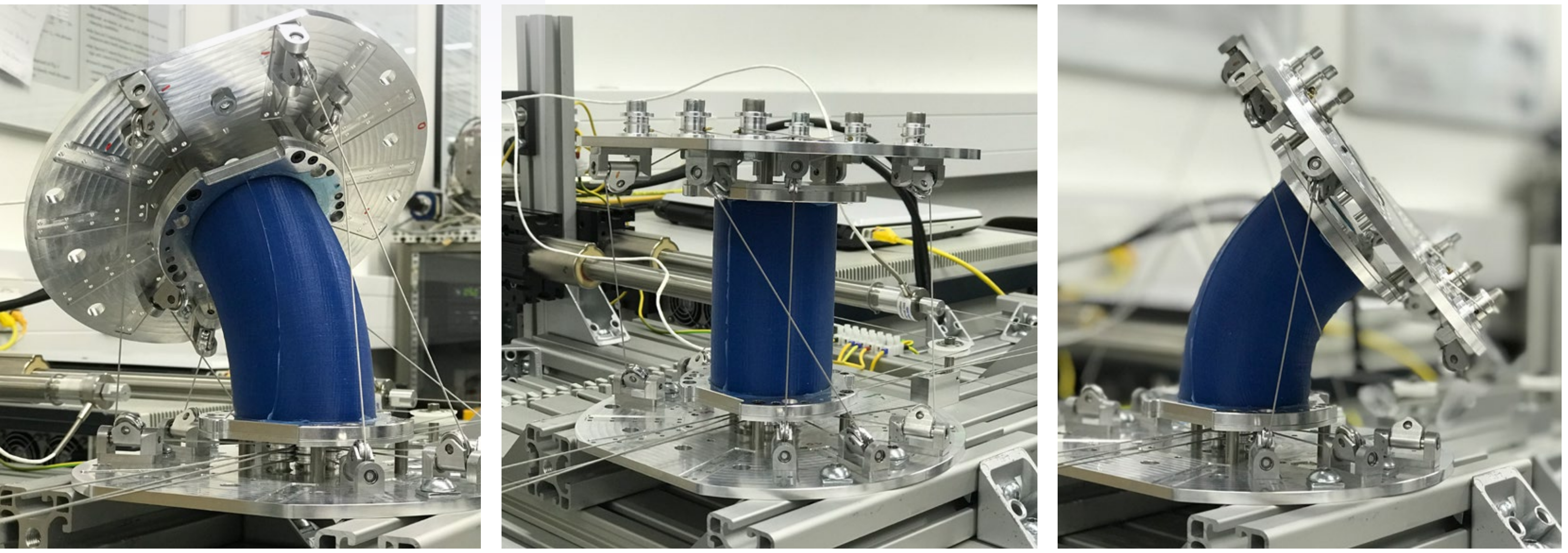
On microscopic scales, elastic materials are made up of molecular building blocks whose arrangements can readily deform when forces are applied, and return to their original states when those forces are removed. The idea seems simple at first glance – but in reality, there are many different mechanisms by which elastic structures can deform. Today, the latest advances in materials science have led to highly deformable materials with particularly intriguing structures.

Among these new products are elastic metamaterials: structures that are artificially engineered to display mechanical properties that cannot be found in nature. Intriguingly, the physical properties of metamaterial building blocks bear very little resemblance to the material's overall behaviour. Instead, they collectively work together to bring about far more complex behaviour on macroscopic scales. In his research at the University of Stuttgart, Dr Simon Eugster explores how mathematical descriptions of highly deformable elastic metamaterials can be used to learn more about their application in real-world scenarios.

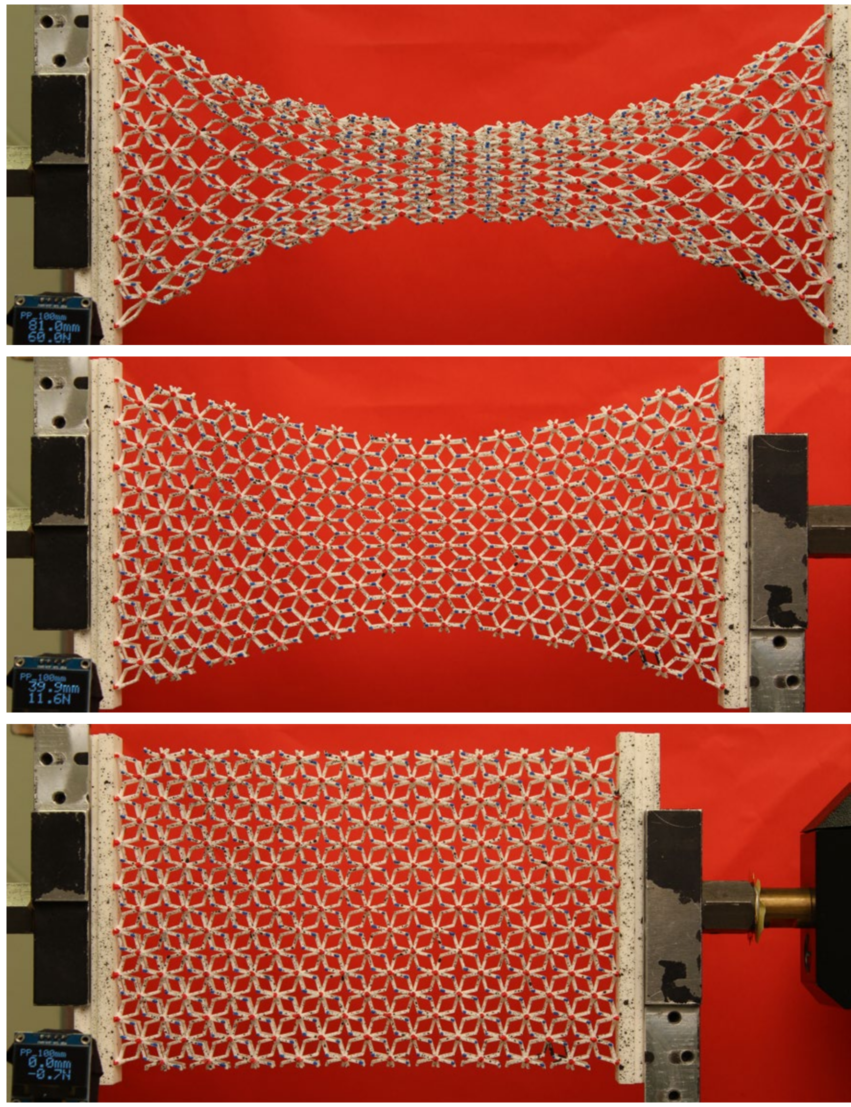
ELASTIC DEFORMATION AND PANTOGRAPHS

Traditionally used by artists, architects, and mathematicians alike, pantographs are instruments made from mechanical networks of beams and pivots, which allow their users to draw both enlarged and miniaturised copies of original images. A key feature of these systems is that their mechanical behaviour as a whole doesn't closely reflect that of their individual beams. Currently, 3D printing technologies are capable of producing pantographic building blocks on centimetre scales.

Recently, materials physicists have aimed to produce similar mechanisms on microscopic scales, for which a pantographic accordion structure is particularly desirable, as it allows metamaterials to readily compress and decompress. For now, limitations in 3D printing technologies mean that such advanced structures can't yet be fabricated in the lab. Yet as researchers await these highly advanced techniques, mathematical formulations are being developed which can describe microscopic pantographic materials as being completely uniform, with no visible influences from individual beams or pivots.



Testbed for tendon-driven continuum joint modules.



Elastic deformation of a sheet with pantographic beams as microstructure.

Elastic metamaterials are artificially engineered to display mechanical properties that cannot be found in nature.

'Pantographic metamaterials can endure large elastic deformation, whose mechanics depend only moderately on their constituting material,' Eugster explains. 'They have proven to be extremely resilient when it comes to failure.' In a continuum description, the first order approximation of the change of length between two and the change of angle between three neighbouring points are determined by the first gradient of the placement map between an undeformed reference configuration and the deformed configuration. These two deformation measures are called *stretch* and *shear*.

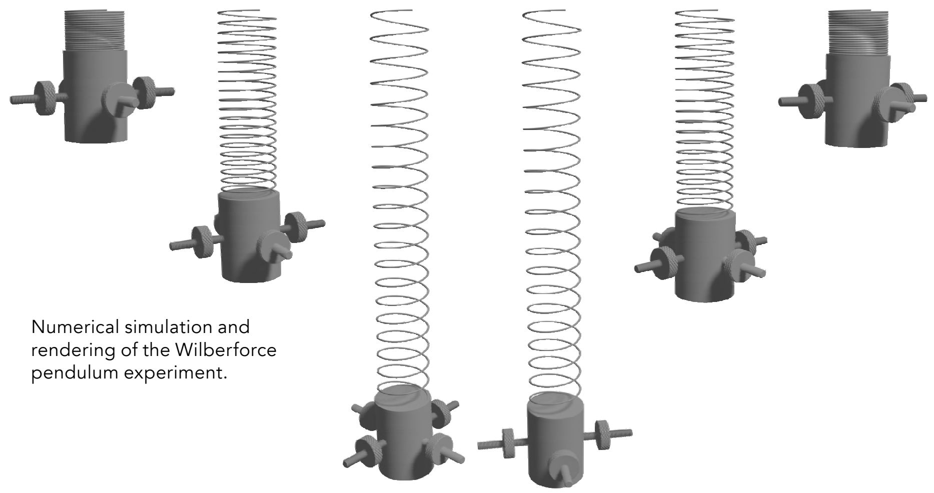
Taking the second gradient of placement into account, one can describe more non-local deformation measures such as curvatures or the change of stretch when conceptually moving along a thought curve within the material. The deformation of pantographic metamaterials is not only influenced by the first but also by the second gradient of the placement map.

The concept of pantographic materials has now settled a key question posed by the very existence of second gradient metamaterials: is there a micro-structured material whose deformation can have a mathematical description, in which only the second gradient of placement is of relevance? The very fact that this idea can be proven to be correct, without even constructing a metamaterial in the real world, has important consequences.

'The striking fact about these materials is that the idea of a pantographic microstructure was purely academic until recently,' Eugster illustrates. By developing advanced new mathematical theories, Eugster and researchers like him are paving the way for entirely new classes of fascinating elastic materials, which would have been practically impossible to develop through trial-and-error synthesis in the lab. In turn, his team's ideas are also leading to new improvements to theories that have already been well established.

IMPLICATIONS FOR BEAM THEORY

The physical concepts used to describe the elastic deformation of long, slender beams has existed for hundreds of years. They were first laid out by Swiss



Numerical simulation and rendering of the Wilberforce pendulum experiment.

mathematician Leonhard Euler in the 18th century, and described in precise detail by brothers Eugène and François Cosserat, in a seminal work on deformable bodies published in 1909. According to Euler and the Cosserat brothers, we can essentially think of beams as one-dimensional structures, with each point along a central line being associated with its own 2D cross-section. 'In classical beam theories, the placement of a slender structure is described by a curve – the centreline, moving in space together with rigid cross sections attached to each point of the curve,' Eugster describes.

To this day, the theory remains instrumental to engineers, in describing how beams with different-shaped cross-sections will deform when heavy loads are applied to them. Yet in more recent research, where deformable beams are used as the building blocks of far larger metamaterials, existing theories aren't yet sufficient to fully describe what is happening. To address the issue, Eugster and his colleagues take the mathematics of beam theory all the way back to Euler's very first descriptions.

In Euler's time, the physical theories required to describe the mechanics of deforming 3D bodies didn't yet exist. To explore the physical characteristics of deforming beams, he was essentially limited to 1D theories, which were far easier to solve in an analytical way. As a result, although connections between beam theory and 3D elasticity can already be described for very small deformations, the descriptions we continue to use today are not directly linked to the full 3D mechanics of deforming materials. In their research, Eugster and his colleagues are developing new mathematical tools to explore how the mechanics of large deformations to 3D bodies relate to several aspects of beam deformation. By updating numerical theories to predict both the static and dynamic behaviours of beams, the team hopes that their findings will come to shape the application of deforming beams within real-world metamaterials, along with other cutting-edge areas of research and engineering.

LIFELIKE DEFORMABLE ROBOTS

Soft robotics is a field that involves robots made from highly compliant materials, and has rapidly gained interest in recent years. Researchers are hoping to use them in the near future, in an increasingly diverse array of applications – from assistance in delicate surgical procedures, to the exploration of deep ocean environments. One particularly important challenge facing the field is the need to produce 'tendon-driven robots' – which aren't driven by linear or rotating motors, but by cables which can be coiled and unwound, varying their length.

In his research, Eugster considers how updated mathematical theories could help us to describe the behaviours of elastically deformable cables in a simple mechanical system. 'We are developing a modular elastic mechanism composed of a silicone block between two plates,' he describes. 'The two plates are connected by several actuated tendons, which cause the silicone block to bend, twist, shear, and compress.' While this task would be an enormous challenge using the mathematics developed by Euler, Eugster's team hope that the numerical tools they have developed will provide a full description of the overall behaviour of such a mechanism.

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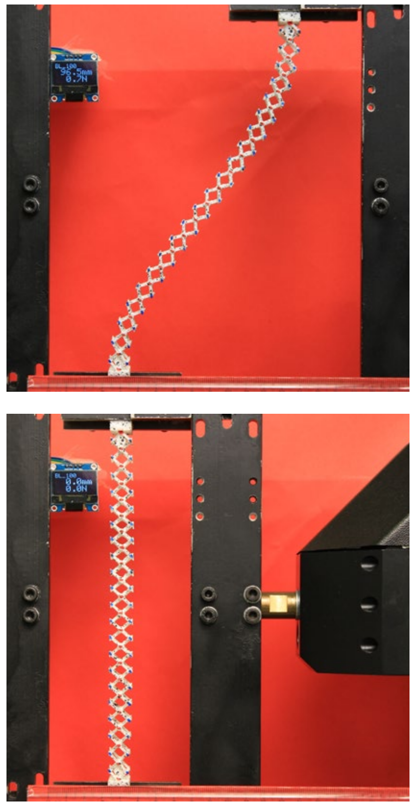

Shear test of a pantographic beam.

By drawing on their updated descriptions, Eugster and his colleagues can gain an in-depth picture of how their system of connected plates will behave in a variety of different scenarios. 'With such models, we can not only design these mechanisms, we can also use them to estimate the placement of the elastic components,' says Eugster. 'With an appropriate model, and the measurements of how much tendon has been coiled up, we can predict the entire shape of the mechanism.'

In turn, the team are able to accurately determine how the top plate will be positioned and oriented when the tendons connecting them are actuated in different ways. Such an in-depth understanding is crucial when designing soft robots that can be easily controlled by their users – particularly in situations where sensory information is limited, such as when the robot cannot be observed by external cameras. With the ability to adapt the motions and orientations of soft robots in this way, they will be far better suited to operating in widely varying environments, where conditions can be highly unpredictable.

SHAPING APPLICATION WITH THEORY

In their future research, Eugster and his colleagues will continue to explore how updated theories of highly deformable elastic structures can lead to more advanced applications in metamaterials and soft robotics. Their work is turning the traditional order of scientific discovery on its head: developing new theories to enable the design of material structures that may have seemed impossible just a few years ago. As new applications emerge in the real world, the advances enabled by these discoveries could soon come to play an increasingly important role in our everyday lives.

Behind the Research

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Research Objectives

Developing advanced mathematical theories to describe highly deformable elastic structures and materials.

Detail

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Bio
Simon R Eugster is Senior Lecturer at the Institute for Nonlinear Mechanics (INM), University of Stuttgart, Germany, since 2020. He received his MSc and PhD degree from ETH Zurich, Switzerland, in 2009 and 2014. In 2014, he became Lecturer at the INM.

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Personal Response

How close are you to developing real pantographic materials in the lab?

" On the centimetre scale these prototypical structures, which can be described by planar theories, do already exist. On the same scale, we are currently working on structures with emerging spatial behaviour. On smaller scales, promising fabrication techniques – for instance stereolithography – are available. I am optimistic that with the right collaboration, materials with a pantographic microstructure on micrometre scales can be obtained within the next ten years. "

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